ALGEBRA PROPERTIES

ARITHMETIC PROPERTIES

ASSOCIATIVE $a(bc) = (ab)c$

COMMUTATIVE $a + b = b + a$ and $ab = ba$

DISTRIBUTIVE $a(b + c) = ab + ac$

ARITHMETIC OPERATIONS EXAMPLES

$ab + ac = a(b + c)$

$a\left(\frac{b}{c}\right) = \frac{ab}{c}$

$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$

$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

$\frac{a - b}{c - d} = \frac{b - a}{d - c}$

$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{ab + ac}{a} = b + c, a \neq 0$

$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$

QUADRATIC EQUATION

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXPONENT PROPERTIES

$a^n a^m = a^{n+m}$

$(a^n)^m = a^{nm}$

$(ab)^n = a^n b^n$

$a^{-n} = \frac{1}{a^n}$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

$a^0 = 1, a \neq 0$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\frac{1}{a^{-n}} = a^n$

$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$

PROPERTIES OF INEQUALITIES

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $a/c < b/c$

If $a < b$ and $c < 0$ then $ac > bc$ and $a/c > b/c$

PROPERTIES OF COMPLEX NUMBERS

$i = \sqrt{-1}$

$i^2 = -1$

$\sqrt{-a} = i\sqrt{a}, \quad a \geq 0$

$(a + bi) + (c + di) = a + c + (b + d)i$

$(a + bi) - (c + di) = a - c + (b - d)i$

$(a + bi)(c + di) = ac - bd + (ad + bc)i$

$(a + bi)(a - bi) = a^2 + b^2$

$|a + bi| = \sqrt{a^2 + b^2}$

$\overline{(a + bi)} = a - bi$

$\overline{(a + bi)}(a + bi) = |a + bi|^2$

$\frac{1}{(a + bi)} = \frac{(a - bi)}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2}$

RADICAL PROPERTIES

$a, b \geq 0$ for even n

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

LOGARITHM PROPERTIES

if $y = \log_b x$ then $b^y = x$

$\log_b b = 1$ and $\log_b 1 = 0$

$\log_b b^x = x$

$b^{\log_b x} = x$

$\log_a x = \frac{\log_b x}{\log_b a}$

$\log_b(x^r) = r \log_b x$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

COMMON FACTORING EXAMPLES

$x^2 - a^2 = (x + a)(x - a)$

$x^2 + 2ax + a^2 = (x + a)^2$

$x^2 - 2ax + a^2 = (x - a)^2$

$x^2 + (a + b)x + ab = (x + a)(x + b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x + a)^3$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

ABSOLUTE VALUE

$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$

$|a| = |-a|$

$|a| \geq 0$

$|ab| = |a||b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a + b| \leq |a| + |b|$

COMPLETING THE SQUARE

$ax^2 + bx + c = a(\ldots)^2 + \text{constant}$

1. Divide by the coefficient a.
2. Move the constant to the other side.
3. Take half of the coefficient b/a, square it and add it to both sides.
4. Factor the left side of the equation.
5. Use the square root property.
6. Solve for x.

CALCULUS

DERIVATIVES AND LIMITS

DERIVATIVE DEFINITION

$$\frac{d}{dx}(f(x)) = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

BASIC PROPERTIES

$$(cf(x))' = c(f'(x))$$

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$\frac{d}{dx}(c) = 0$$

MEAN VALUE THEOREM

If f is differentiable on the interval (a,b) and continuous at the end points there exists a c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

PRODUCT RULE

$$(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

CHAIN RULE

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

COMMON DERIVATIVES

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$$

CHAIN RULE AND OTHER EXAMPLES

$$\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1}f'(x)$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\sin[f(x)]) = f'(x)\cos[f(x)]$$

$$\frac{d}{dx}(\cos[f(x)]) = -f'(x)\sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)]) = f'(x)\sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$

$$\frac{d}{dx}(\tan^{-1}[f(x)]) = \frac{f'(x)}{1+[f(x)]^2}$$

$$\frac{d}{dx}(f(x)^{g(x)}) = f(x)^{g(x)}\left(\frac{g(x)f'(x)}{f(x)} + \ln(f(x))\,g'(x)\right)$$

PROPERTIES OF LIMITS

These properties require that the limit of $f(x)$ and $g(x)$ exist

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \text{ if } \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$$

LIMIT EVALUATION METHOD – FACTOR AND CANCEL

$$\lim_{x\to -3}\frac{x^2-x-12}{x^2+3x} = \lim_{x\to -3}\frac{(x+3)(x-4)}{x(x+3)} = \lim_{x\to -3}\frac{(x-4)}{x} = \frac{7}{3}$$

L'HOPITAL'S RULE

$$\text{If } \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty} \text{ then } \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

LIMIT EVALUATIONS AT +-∞

$$\lim_{x\to\infty} e^x = \infty \text{ and } \lim_{x\to-\infty} e^x = 0$$

$$\lim_{x\to\infty} \ln(x) = \infty \text{ and } \lim_{x\to 0^+} \ln(x) = -\infty$$

$$\text{If } r > 0 \text{ then } \lim_{x\to\infty}\frac{c}{x^r} = 0$$

$$\text{If } r > 0 \text{ \& } x^r \text{ is real for } x < 0 \text{ then } \lim_{x\to-\infty}\frac{c}{x^r} = 0$$

$$\lim_{x\to\pm\infty} x^r = \infty \text{ for even } r$$

$$\lim_{x\to\infty} x^r = \infty \text{ \& } \lim_{x\to-\infty} x^r = -\infty \text{ for odd } r$$

CALCULUS

INTEGRALS

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$

FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

where f is continuous on $[a,b]$ and $F' = f$

INTEGRATION PROPERTIES

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)dx$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)dx \pm \int_a^b g(x)\,dx$$

$$\int_a^a f(x)dx = 0 \text{ and } \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)dx = \int_a^c f(x)\,dx$$

COMMON INTEGRALS

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$$

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int \ln(x)\,\,dx = x\ln(x) - x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \cos x\,\,dx = \sin x + C$$

$$\int \sin x\,\,dx = -\cos x + C$$

$$\int \sec^2 x\,\,dx = \tan x + C$$

$$\int \sec x \tan x\,\,dx = \sec x + C$$

$$\int \csc x \cot x\,\,dx = -\csc x + C$$

$$\int \csc^2 x\,\,dx = -\cot x + C$$

$$\int \tan x\,\,dx = \ln|\sec x| + C$$

$$\int \sec x\,\,dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{a^2+u^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,dx = \sin^{-1}\left(\frac{u}{a}\right) + C$$

APPROXIMATING DEFINITE INTEGRALS

Left-hand and right-hand rectangle approximations

$$L_n = \Delta x\sum_{k=0}^{n-1} f(x_k) \qquad R_n = \Delta x\sum_{k=1}^{n} f(x_k)$$

Midpoint Rule

$$M_n = \Delta x\sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

Trapezoid Rule

$$T_n = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n)\right)$$

TRIGNOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EXPRESSION EVALUATION	IDENTITY USED
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\,d\theta$	$\sqrt{a^2 - a^2\sin^2\theta}$ $= a\cos\theta$	$1 - \sin^2\theta$ $= \cos^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\,d\theta$	$\sqrt{a^2\sec^2\theta - a^2}$ $= a\tan\theta$	$\sec^2\theta - 1$ $= \tan^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\,d\theta$	$\sqrt{a^2 + a^2\tan^2\theta}$ $= a\sec\theta$	$1 + \tan^2\theta$ $= \sec^2\theta$

APPROXIMATION BY SIMPSON RULE FOR EVEN N

$$S_n = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)$$

INTEGRATION BY SUBSTITUTION

$$\int_a^b f(g(x))\,g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$ and $du = g'(x)dx$

INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du \quad \text{where } v = \int dv$$

or

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$